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VDI 1978, no. 3

О СТАНДАРТАХ ЕМКОСТИ ЭЛЛИНИСТИЧЕСКОГО ХЕРСОНЕСА

Углубленное изучение материала по экономике и торговле античных государств требует все более активного привлечения сведений по метрологии. Важность метрологических исследований для изучения этих проблем была отмечена ещё в прошлом веке Г. Ниссеном [1]. К сожалению, этой теме до сих пор уделяется мало внимания. Лишь вышедшая недавно работа И. Б. Брашинского посвящена вопросам методики определения ёмкостей керамической тары [2]. Так как автор достаточно полно охарактеризовал степень изученности этой проблемы, мы не будем на ней подробно останавливаться. Отметим, только, что основное внимание исследователей было нацелено на изучение весовой и линейной метрологии Ольвии, Пантикапея и Херсонеса, преимущественно весовых стандартов.

[1] H. Nissen, Griechische und römische Metrologie, Handbuch der klassischen Altertumswissenschaft, I[2], München, 1892, стр. 850 сл.

[2] И. Б. Брашинский, Методика изучения стандартов древнегреческой керамической тары, СА, 1976, № 3, стр. 87–104.

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Впервые метрологические изыскания по северопричерноморским весовым системам были предприняты А. Л. Бертье-Делагардом, выводы которого позднее дополнены А. Н. Зографом и Д. Б. Шеловым [3]. Среди монетно-весовых систем зафиксированных в Северном Причерноморье, выделяется родосско-хиосская, персидская, эгинская, эвбейская и другие [4]. Эгинская система доминировала в Ольвии и Пантикапее [5], родосско-хиосская — в Херсонесе (Зограф, ук. соч., стр. 147). Здесь выведена также персидская, а в Пантикапее и Ольвии — эвбейская система. По мнению Л. И. Чуистовой, можно выделить группу гирь и собственной херсонесской системы с весом мины 207 г [6]. Но так как она исходит из неверного определения буквенного обозначения на каменной гире (39 вместо 19), этот вывод нельзя признать удачным. Кроме того, метод взвешивания металлических гирь, который применила Л. И. Чуистова, неверен [7], поэтому выводы автора нельзя использовать.

При характеристике линейных стандартов Боспора Э. О. Берзин отмечает, что здесь, как и в Ольвии, применялась аттико-эвбейская (ионийская) система [8]. Н. И. Болотин также выделяет в Пантикапее эвбейскую (ионийскую) систему, но в Ольвии он считает основной эгинскую [9], но И. Б. Брашинский отметил, что черепицы этого центра, а также Херсонеса изготовляли по аттическому (фут в 326 мм) стандарту [10]. Частично разработан вопрос о линейных мерах Херсонеса. С. Ф. Стржелецкий определил, что единицей измерений, с помощью которой размежеваны наделы Херсонеса на Гераклейском полуострове, была оргия, близкая или равная аттической [11]. А. Н. Щеглов, изучая памятники хоры Херсонеса в Северо-Западном Крыму, пришел к аналогичному выводу [12].

Стандартам емкости указанных центров до сих пор не было посвящено специального исследования, Б. Н. Граков, который начал над этим работать и измерил емкости 150 амфор, завершить этот труд не успел. И. Б. Зеест и В. В. Борисова приводят емкости некоторых типов амфор, не определяя их стандарты [13]. Недавно в связи с рассмотрением типов амфор коснулись вопроса стандарта емкости амфор В. И. Кац и С. Ю. Монахов [14]. Определение стандартных величин емкости может пролить свет на многие стороны экономической жизни древнего города, о чем подробно говорит И. Б. Брашинский (ук. соч., стр. 87—89).

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[3] А. Л. Бертье-Делагард, Относительная стоимость монетных металлов на Боспоре и Борисфене в половине IV в. до н. э., Нумизматический сборник, т. I, М., 1911; А. Н. Зограф, Античные монеты, МИА, 16, 1951; Д. Б. Шелов, Монетное дело Боспора IV—II вв. до н. э., М., 1956.

[4] Зограф, ук. соч., стр. 125, 127.

[5] Шелов, ук. соч., стр. 73, 83.

[6] Л. И. Чуистова, Античные и средневековые весовые системы, имевшие хождение в Северном Причерноморье, в сб. «Археология и история Боспора», т. II, Симферополь, 1962.

[7] Н. Л. Грач, Свинцовые гири из Нимфея и некоторые вопросы боспорской весовой метрологии, «Труды Гос. Эрмитажа», т. XVII, Л., 1976, стр. 194.

[8] Э. О. Берзин, О линейных мерах Боспора, СА, XXVI, 1956, стр. 227—235.

[9] N. Bolotin, Antike Maße in ihrer Bedeutung für Geschichte und Kunst, «Das Altertum», Bd. 15, 1959, Ht. 4, стр. 216 сл.

[10] И. Б. Брашинский, Стандарты линейных мер в керамическом производстве Синопы, ИКАМ, М., 1977, стр. 35 сл.

[11] С. Ф. Стржелецкий, Клеры Херсонеса Таврического, ХСб., VI, Симферополь, 1961, стр. 56—57.

[12] А. Н. Щеглов, Северо-Западный Крым в античную эпоху, Автореф. канд. дисс., Л., 1971, стр. 14; он же, Жилой дом эллинистического Калос Лимена, «Художественная культура и археология античного мира», М., 1976, стр. 235.

[13] И. Б. Зеест, Керамическая тара Боспора, МИА, 83, 1960, стр. 75 сл.; В. В. Борисова, Керамические клейма Херсонеса и классификация херсонесских амфор, НЭ, XI, 1974, стр. 102, 111.

[14] В. И. Кац, С. Ю. Монахов, Амфоры эллинистического Херсонеса, «Античный мир и археология», вып. III. Саратов, 1977, стр. 102 сл. Соглашаясь с выводами авторов о стандартах емкости, вычисленных ими эмпирическим путем, нужно отметить, что по количеству использованных для анализа амфор (10 экз.) гистограмму емкостей строить нельзя. Для получения наиболее вероятного вывода по гистограмме число статистических однородных данных (наблюдений) должно приближаться к сотне (см. Е. С. Вентцель, Теория вероятности, М., 1962, стр. 192).

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В настоящей статье автор излагает результаты работы по определению стандартов емкости херсонесских амфор эллинистического периода, которые она попыталась установить с применением математического метода вычисления емкости амфор. Так как методика определения стандартов емкости сочетает в себе линейные и фактические измерения емкостей, необходимо более подробно рассмотреть наиболее распространенные системы линейных мер. Известно, что основой для образования мер емкости и веса служили линейные меры [15], при этом один кубический фут соответствует метрическому выражению веса таланта, полтора фута — весу метрета, два фута — весу медимна.

Наиболее употребительным был фут в 295,7—296 мм, образованный от длины персидского или финикийского локтя в 443, 55—444 мм и составлявшего 4/5 вавилонского большого локтя в 555 мм, и фут в 297 мм от локтя в 445,5 мм образованного от древневавилонского локтя в 495 мм [16]. В литературе этот фут, равный 296—297 мм, часто называется аттическим. Известен также фут в 326—328 мм от локтя в 495 мм (там же стр. 857). Дерпфельд называл его эгинским и считал наиболее употребительным в Афинах [17]. В работах последних лет за футом 296 мм укрепилось название короткого, или ионийского, а за футом 326 мм — длинного, дорийского, или аттического [18]. В связи с этим необходимо заметить, что исследователи, говоря об употреблении в Херсонесе эвбейской системы линейных мер, имели в виду фут в 296 мм, т. е. ионийский.

Для выявления стандартов емкости нами измерено 45 амфор херсонесского производства, хранящихся в фондах Херсонесского и других музеев, и кроме того, использованы данные по емкостям, приведенные в работе В. В. Борисовой [19]. Емкость целых и реставрированных амфор определена эмпирически, при этом каждая амфора наполнялась водой или зерном до венчика горла. Емкость всех амфор вычислена и математическим путем.

Первый опыт математического вычисления емкости тары был проделан нами на примере синопского пифоса с цифровыми метками [20]. Полный объем пифоса вычислен по известной в математике формуле $V = \frac{\pi}{3} \sum \left(R_j^2 + R_j \cdot R_j + R_j^2\right) \Delta h$ для приближенного вычисления объемов тел вращения. Смысл этой формулы состоит в том, что тело вращения по оси разбивается на ряд элементарных тел вращения, в данном случае — усеченных конусов. Сумма объемов этих усеченных конусов дает полный объем тела вращения. Емкость пифоса, вычисленная по этой формуле, составила 1065 ± 65 л. Если вычислить емкость по формуле Герона для пифоидных тел, использованной М. Лэнг [21] для вычисления объема гипотетического фасосского пифоса, то она составит

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[15] Подробнее об этом см. N i s s s e n, ук. соч., стр. 849; Н. Т. Б е л я е в, О древних и нынешних русских мерах протяжения и веса, Seminarium Kondakovianum, v. I, Prague, 1927, стр. 247 сл.

[16] N i s s e n, ук. соч., стр. 861, 857.

[17] W. D ö r p f e l d, Metrologische Beiträge, «Mittheilungen des Deutschen Archäologischen Instituts in Athen», XV, 1890, стр. 167.

[18] M. L a n g, A new inscription from Thasos: Specifications for a Measure, BCH, 76, 1952, стр. 22—28; W. D i n s m o o r, The Basis of Greek Temple Design: Asia Minor, Greece, Italy, Atti del settimo Congresso Internationale di archeologia classica, vol. I, Roma, 1961, стр. 358 сл. у Лэнг значения футов 296 и 326 мм, у В. Динсмура — 293,9 и 326,55 мм. В своих дальнейших расчетах мы будем пользоваться ионийским футом 296 мм и дорийским футом 326 мм.

[19] Б о р и с о в а, ук. соч., стр. 102—104. Измерения амфор из раскопок Панское-I проведены С. Ю. Монаховым.

[20] Г. М. Н и к о л а е н к о, Метки на античных пифосах, «Херсонес Таврический, Ремесло и культура», Киев, 1974, стр. 29.

[21] L a n g, ук. соч., стр. 28. Формула имеет вид: $V = 11/14 \cdot \left(\frac{D\text{ устья} + D\text{ тулова}}{2}\right)^2 \cdot h$, где h, по мнению М. Лэнг, глубина пифоса. Известна также формула Герона для вычисления емкости сфероидного пифоса: $V = 11/21 \left(\frac{D\text{ у.} + D\text{ т.}}{2}\right)^2 \cdot h$. Об этом см. Metrologicorum scriptorum reliquiae (ed. Fr. Hultsch), Lipsiae, 1864, стр. 202 слл., № 19—20.

984,25 л. Разницу в вычислениях можно объяснить тем, что при использовании современной формулы возникают погрешности при вычерчивании профиля сосуда, что обусловлено практическим отклонением формы пифоса от идеальной. Эти погрешности составляют примерно 5% от объема сосуда. Определить погрешности при вычислениях по формуле Герона, полученной, вероятнее всего, эмпирическим путем, можно при наличии большого статистического материала, которого к настоящему времени мы еще не имеем. Кроме того, М. Лэнг считает, что один из диаметров, приведенных в формуле Герона,— внутренний диаметр тулова. Но этот диаметр трудно измерить практически. Гораздо удобнее было мастеру проверить своё изделие, измеряя наружный диаметр тулова и внутренний диаметр устья. Несомненно, так же поступал мастер, когда проверял размеры амфоры[22]. Если, подтверждая наши рассуждения, вычислить емкость пифоса, подставив в формулу Герона значение наружного диаметра тулова, то она составит 1027,5 л. Этот результат довольно близок к значению объема, полученного по формуле суммы объемов усеченных конусов и соответствует 20 медимнам или 26 метретам или 30 аттическим куб. футам или 40 ионийским куб. футам.

Этой же формулой (сумма усеченных конусов) мы воспользовались при вычислении емкости амфор. В результате промеров можно выделить стандарты амфор, емкость которых колеблется в следующих пределах:

Конец IV — середина III в. до н. э.[23]

первый — 31,5—32,5 л; *второй* — 19,40—19,60 л; *третий* — 17,20—17,50 л; *четвертый* — 14,50—14,70 л; *пятый* — 4,80—5,00 л. Предположительно выделяются стандарты: *шестой* — 30,20—30,30 л; *седьмой* — 27,20—27,50 л; *восьмой* — 22,50—22,70 л; *девятый* — 9,60—9,80 л.

Середина III—II в. до н. э.

восьмой — 22,50—22,70 л; *третий* — 17,20—17,50 л; *девятый* — 9,60—9,80 л; *пятый* — 4,80—5,00 л. Предположительно выделяются стандарты: *второй* — 19,40—19,60 л и *десятый* — 8,50—8,60 л.

Стандарты 1, 2, 4, 9 и 5 соответственно равны 10; 6; 4; 5; 3; 1,5 эвбейским хоям по 3,2 л или 11,5; 7; 5; 3,5; $1^2/_3$ хиосским хоям по 2,8 л. Стандарты 3 и 10 равны 8 и 16 хойникам по 1,08 л или 1/3 и 1/6 эвбейского медимна, а также 6 и 3 хиосским хоям. Стандарты, определяемые предположительно, равны: шестой стандарт — 1 понтийскому марису в 30,31 л; седьмой — 8,5 эвбейским хусам или $9^1/_3$ хиосских хусов; 6 понтийским аддиксам или эгинским хоям по 4,55 л; восьмой — 22,75 л, то есть 7 эвбейским или 8 хиосским хоям, а также 5 понтийским аддиксам. Стандарт четвертый равен также одной понтийской гидрии в 14,58 л, а стандарты пятый и девятый соответственно 1/3 и 2/3 понтийской гидрии.

Вероятно, наиболее распространенным стандартом конца IV — середины III в. до н. э. был второй стандарт — 19,40—19,60 л, так как амфоры такой емкости составляют 50% от общего числа амфор указанного времени. Для середины III—II в. до н. э. наиболее характерным был, вероятно, стандарт пятый — 4,80—5,00 л (50% от общего числа амфор этого времени). Стандарт третий — 17,20—17,50 л составляет 25% от общего числа амфор IV—II вв. до н. э. и, возможно, был распространен на протяжении этого периода.

Так как все стандарты содержат определенное число эвбейских хоев, то возможно предположение, что при изготовлении амфоры мастер пользовался эвбейскими стандартами емкости, причем такими, которые легко пересчитать в стандарты других систем.

На основании проведенных вычислений можно высказать гипотезу о том, что в Херсонесе одновременно существовали стандартные фракционные сосуды, а также, что основной стандарт был различным в разные периоды. Это предположение опирается на выводы В. Грэйс, которая на основании обработки значительного количества материала пришла к заключению о стандартности остродонных амфор, существовании не-

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[22] Брашинский, ук. соч., стр. 97, прим.

[23] Датировка керамического материала дана по указанной работе В. В. Борисовой.

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скольких стандартов в одном центре одновременно и изменении стандарта на протяжении определенного отрезка времени в одном центре [24]. Разнообразие стандартов и значительная емкость амфоры основного стандарта (19,40—19,60 л), возможно, объясняется тем, что Херсонес в конце IV — начале III в. до н. э. производил значительное количество вина на продажу. Уменьшение стандарта емкости в III—II вв. до н. э., вероятно, обусловлено тем, что в результате войн со скифами город теряет значительную часть своей территории, а вследствие этого сокращается производство вина.

Известно, что Херсонес в выпусках тары подражал типам Гераклеи Понтийской и Синопы [25]. Н. А. Лейпунская отметила однообразие амфор гераклейского производства внутри типа, что обусловлено, по мнению автора, стандартизированным характером керамического производства и торговли Гераклеи [26]. Основные линейные параметры амфор этого центра колеблются в пределах: высота сосуда — 61—75,5 см, диаметр тулова — 22—29 см, диаметр устья — 8—10 см [27]. Линейные размеры амфор херсонесского производства, подражающих формам гераклейских, укладываются в эти же рамки. Нужно отметить также, что значительное число херсонесских амфор близки по размерам синопским, фасосским и пантикапейским.

Производство керамической тары, вероятно, осуществлялось по указанным государственным эталонам [28], и совпадение размеров амфор херсонесского производства с амфорами перечисленных центров, возможно, свидетельствует о том, что Херсонес, являвшийся значительным торговым центром, стремился унифицировать стандарты мер, что упрощало совершение торговых операций. Исследователями отмечен важный факт: промеры амфор различных типов дают определенное устойчивое однообразие метрических признаков в амфорах одного типа, а именно: высоты, глубины, диаметра тулова, устья, высоты верхней части (до линии диаметра тулова), соотношение высоты нижней части амфоры и полной ее высоты [29].

Работа по определению пропорциональной зависимости между различными частями амфоры предпринята Н. А. Лейпунской. Ею высчитана пропорциональная зависимость между высотой нижней части амфоры и полной высотой. Идеальное частное этой пропорции — 0,618 [30], т. е. 3/5 высоты амфоры. Аналогичные расчеты, проведенные на херсонесских амфорах, подтверждают этот результат. Отсюда, отношение высоты верхней части амфоры к полной ее высоте — 0,368, т. е. 2/5 высоты амфоры. Кроме того, установлено, что отношение высоты горла к полной высоте у амфор херсонесского производства колеблется в пределах 0,200—0,247, т. е. 1/5—1/4 высоты амфоры, а отношение диаметра горла к диаметру тулова — 0,210—0,240.

По линейным параметрам херсонесские амфоры определенных емкостей можно распределить на несколько групп: *первая* — емкость 5 л, высота 50—54 см, диаметр тулова 21—24 см; *вторая* — емкость 10—11 л, высота 68—70 см, диаметр тулова 24—28 см; *третья* — емкость 14—15 л, высота 69—71 см, диаметр тулова 27—28 см; *четвертая* — емкость 16,5—19,5 л, высота 70—71 см, диаметр тулова 29—30 см; *пятая* — емкость 22—27 л, высота 70—71 см, диаметр тулова 30—36 см; *шестая* — емкость 30—32 л, высота 69—72 см, диаметр тулова 35,5—37 см, *седьмая* — емкость 22—27 л, высота 75—88 см, диаметр тулова 53—57 см.

Высота амфор первой группы составляет 3/4 высоты амфор второй — шестой групп, диаметры тулова амфор первой — шестой групп составляют соответственно 1/3, 2/5, 2/5, 2/5, 1/2, 1/2 высоты амфор второй — шестой групп. Диаметр горла всех групп амфор 8—10 см, что составляет 1/9 высоты амфор второй — шестой групп. Глубина

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[24] V. Grace, Standard Pottery Containers of the Ancient Greek World, «Hesperia», Suppl. VIII, 1949, стр. 175.

[25] Борисова, ук. соч., стр. 101.

[26] Н. А. Лейпунская, О стандартах гераклейской амфорной тары, «Ольвия», Киев, 1975, стр. 130.

[27] Там же, стр. 127.

[28] Брашинский, Методика изучения..., стр. 90.

[29] Там же, стр. 93; Лейпунская, ук. соч., стр. 131.

[30] Н. О. Лейпунська, Принцип пропорціональності в античных керамічних виробах, «Археологія», 15, 1975, стр. 25 сл.

амфор первой группы составляет 3/5 высоты второй — шестой групп, глубина которых 9/10 их высоты.

Исходя из этой таблицы, можно составить ряд соотношений высот и диаметров амфор первой — шестой групп, где за единицу принята высота 69—71 см, т. е. высота амфор наибольшей емкости и наиболее часто повторяющаяся (во всех группах, кроме первой). Этот ряд выглядит так: 1 : 1/9 : 1/5 : 2/5 : 3/5 : 1/3 : 1/2 : 3/4 : 9/10.

Мастеру, конечно, были известны эти соотношения, и для изготовления амфоры определенной емкости ему достаточно иметь один мерный инструмент, где отложены все нужные размеры.

Амфору заданной емкости мастер изготовлял, используя шаблон [31]. Для того, чтобы изготовить шаблон, нужно вычислить теоретическую емкость сосуда. О формулах Герона для расчета теоретической емкости пифосов различной формы уже говорилось. Для расчета емкости амфор такие формулы неизвестны. Но, как показали исследования И. Б. Брашинского, для расчета емкости амфор пифоидного типа можно применять вышеприведенную формулу Герона [32]. Использование этих формул Герона для расчета емкостей всех типов остродонных амфор М. Лэнг и И. Б. Брашинский считают неправомерным [33]. Однако введение коэффициента в формулу Герона делает ее, на наш взгляд, приемлемой для этой цели. Объясним высказанное положение.

Херсонесские амфоры, как и амфоры многих других центров, можно расчленить на геометрические фигуры: цилиндр (горло), усеченный конус (промежуток между линией основания горла и линией максимального диаметра) и конус (нижняя часть амфоры от линии максимального диаметра). Получить емкость амфоры можно, сложив сумму объемов этих фигур, высчитанных по принятым формулам: $V = 1/4\pi d^2$цил $\cdot$ $\cdot h$цил $+ 1/12\pi h$ ус.кон. (d^2цил. $+ d$цил. $\cdot d$тул. $+ d^2$тул.) $+ 1/12\pi d^3$тул. $\cdot h$кон.

Если воспользоваться расчетами соотношения высоты амфоры с высотой отдельных частей и диаметра тулова с диаметром горла и выразить в приведенной формуле диаметр горла и высоты отдельных частей амфоры через эти соотношения, формула будет иметь вид:

$V = 1/4\pi \cdot 0{,}24d^2$тул. $\cdot$ 0,25 hамф. $+ 1/12\pi \cdot 0{,}13h$амф. $\cdot$ ($0{,}24d^2$тул. $+ 0{,}24\,d$ тул. $\cdot$ $\cdot d$тул. $+ d^2$тул.) $+ 1/12\pi d^2$тул. $\cdot$ 0,62hамф. После вычислений получаем результат: $V = 0{,}259d^2h$.

Теперь возьмем формулу Герона: $V = 11/14\dfrac{(d\text{ горла} + d\text{тул.})^2}{2} \cdot h$ и проведем те же действия. Конечный результат выглядит так: $V = 0{,}243d^2h$.

Сравнивая результаты вычислений по этим формулам, можно сказать, что они приближенно равны. Это дает основание использовать формулу Герона для вычислений емкости остродонных амфор.

Преобразуя формулы, мы подставляли максимальное значение соотношений диаметров и высот. Если подставить в эти формулы минимальное значение соотношений, то в конечной формуле получим новое значение коэффициента. Отсюда можно сделать вывод, что значение коэффициента зависит от соотношений диаметров и высот различных частей амфоры, иными словами, от ее пропорций. В конечной формуле это выглядит так: $V = x \cdot d^2h$.

Для амфор херсонесского производства значение x для каждого типа амфор определено по формуле $x = V/d^2h$.

Так как значение коэффициента получено на немногочисленном материале, утверждать его стабильность как для херсонесских, так и для амфор других центров производства пока преждевременно. И все-таки, пытаясь проверить результаты наших вычислений на амфорах Гераклеи Понтийской и Синопы, мы получили удовлетворительный результат, а именно: емкость амфоры, вычисленная эмпирически, почти сов-

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[31] Лейпунская, О стандартах..., стр. 133; Брашинский, Методика изучения..., стр. 90 сл.

[32] Брашинский, Методика изучения..., стр. 92.

[33] Там же, стр. 92—93.

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падает с емкостью, вычисленной по приведенной формуле [34]. Разницу в 200—300 г практически можно не принимать во внимание.

Мы не знаем, как выглядела формула для вычисления тел вращения у греков до Герона, но можем предположить, что греки, хорошо зная математику египтян [35], пользовались их формулами. Известно, что формула объема цилиндра у египтян $(8/9)^2 D^2 H$, в современной интерпретации — $\pi/4\ D^2\ H$. Чтобы вычислить объем тела вращения другой формы, египтяне вводили в формулу объема цилиндра коэффициент [36]. Опираясь на эти сведения, можно сказать, что формула, полученная нами для вычислений объемов, относится к телам (в данном случае амфорам) такой формы, которая требует введения соответствующего коэффициента к формуле объема цилиндра. Иными словами, наша формула может иметь вид: $V = x\pi/4D^2H$.

Анализируя полученный материал, можно сделать следующие выводы:

1. В Херсонесе в эллинистический период выделяются определенные стандарты мер емкости, среди которых для конца IV—середины III в. до н. э. наиболее характерен стандарт 19,40—19,60 л, а для середины III — II вв. до н. э. стандарт 4,80—5,00 л. Колебание стандартов объясняется, вероятно, изменением состояния экономической базы Херсонеса [37].

2. В этот же период в Херсонесе наблюдается определенная унификация стандартов мер. На первый взгляд стандарты как будто разнообразны (в строительстве и архитектуре зафиксировано использование ионийского фута, в торговле — хиосско-родосской и персидской монетно-весовых систем, в керамическом производстве — эвбейских и, возможно, понтийских стандартов емкости). Однако вес емкости, наполненной водой, равный весу хиосско-родосской драхмы (3,88 г), составляет 1/10000, персидской — 1/7000 от эвбейского метрета 38,88 кг, метрический эквивалент которого — 1,5 кубических фута в 296 мм. Иными словами, стандарты емкости и веса можно привести к единому модулю: ионийскому футу в 296 мм.

3. Сравнивая линейные параметры амфор различных центров, можно наблюдать стремление к стандартизации линейных мер этого вида продукции в определенных регионах, в частности, в Херсонесе и торгующих с ним центрах: Гераклее Понтийской, Синопе, Фасосе, Пантикапее.

4. Линейные размеры амфор находятся в пропорциональной зависимости, которая определяется формой амфоры. В формуле эта зависимость выражается коэффициентом.

5. Для математического вычисления емкости остродонных амфор можно применять формулу $x\ (8/9)^2 D^2 H$, используя для каждого типа амфор определенный коэффициент. Такие вычисления дают возможность определить емкость фрагментированных сосудов, что особенно важно для анализа керамического материала, основное количество которого состоит из обломков амфор.

6. Вероятно, не только стандарты мер, но и методы их вычисления греки стремились унифицировать, что имело большое значение в античном производстве и торговле.

Г. М. Николаенко

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[34] Арифметическое значение коэффициента не привожу, так как он получен на недостаточном количестве материала.

[35] М. Я. Выгодский, Арифметика и алгебра в древнем мире, М., 1967, стр. 9.

[36] Г. П. Боев, Вычисление поверхностей и объемов тел вращения у древних египтян, ВДИ, 1950, № 3, стр. 200.

[37] Об этом изменении могут свидетельствовать политические события: потеря владений в Северо-Западном Крыму к началу II в. до н. э., набеги варваров на городскую округу Херсонеса в III—II вв. до н. э., гибель загородных поселков гончаров и виноделов.

1.08

AMPHORA CAPACITY STANDARDS IN HELLENISTIC CHERSONESUS

G. M. Nikolayenko

According to the author's calculations typical capacities are; from the end of the 4th to the mid-3rd century, 19.40—19.60 litres; from the mid-3rd through the 2nd century, 4.80—5.00 litres, corresponding to 1/2 and 1/8 of a Euboean *metretes* of 38.88 litres or to 6 and 1.5 Euboean *choes*, also to 7 and 1 1/2 Chian *choes*. The capacity of a wholly preserved amphora was measured by filling it with sand or water up to the lip. The capacity of restored and fragmentary amphoras was calculated mathematically according to the formula for the sum of the volumes of truncated cones. The linear parameters of an amphora are in a proportional dependence determined by its shape. Given this, one can reproduce the ancient formula for calculating the capacity of bodies in rotation, which will be expressed thus: $x\left(\frac{8}{9}\right)^2 D^2H$, x being the coefficient expressing the correlation of the diameters and heights of the different parts of an amphora, that is, their proportional dependence. By comparing the linear parameters of amphoras from different production centres one can see a tendency towards standardisation of linear measures in this type of product in centres having close commercial ties, in particular in Chersonesus, Pontic Heraclea, Snope, Thasos and Pa nticapaeum.

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About Standards of Capacity of Hellenistic Chersonesos   V.D.I. 1978.3

G.M. Nikolayenko   Vestnik Drevnei Istorii

142 More intensive study of material on the economics and commerce of ancient states demands ever more active application of information about metrology. The importance of metrological researches for study of these problems was already noted in the past by H. Nissen.[1] Unfortunately these topics have not until now received much attention. A work of I.B. Brashinsky which has only recently appeared is devoted to questions of the method of determination of capacities of the ceramic container.[2] Since the author has described sufficiently fully the extent of study of these problems, we will not dwell at length on it. We will note only that the principle attention of investigators was aimed at the study of weight and linear metrology of Olbia, Pantikapaion and Chersonesos, chiefly of weight standards.

143 Metrological researches on north Black Sea area weight systems were undertaken for the first time by A.L. Bertye-Delagard,[3] whose conclusions were later added to by A.N. Zograph and D.B. Shelov.[3] Among monetary-weight systems established in the northern Black Sea, the Rhodian-Chian is distinguished, and the Persian, the Aeginetan, the Euboean, and others.[4] The Aeginetan system predominated at Olbia and Pantakapaion,[5] the Rhodian-Chian in Chersonesos (Zograph op. cit. p. 147). Here also the Persian system has been ascertained, and at Pantikapaion and Olbia the Euboean system. In the opinion of L.I. Chuistova, it is possible to distinguish a group of weights also of a system peculiar to Chersonesos with a mna weight of 207 grams.[6] But since it stems from an inaccurate decipherment of a letter designation on a stone weight (39 instead of 19), that conclusion cannot be admitted successful. Furthermore the method of weighing of metal weights which L.I. Chuistova took is unreliable,[7] and so the conclusions of the author cannot be made use of.

In characterizing linear standards of the Bosphorus, E.O. Berzin observes that here, as also at Olbia, the Attic-Euboic (Ionic) system was in use.[8] N.I. Bolotin also distinguishes at Pantikapaion the Euboic (Ionic) system, but at Olbia he considers the Aeginetan one as basic,[9] but I.B. Brashinsky observed that tiles of that centre, and also of

(143) Chersonesos, they made on the Attic (foot at 326 mm) standard.[10] The question about linear measures of Chersonesos has been partially worked out. S.Ph. Strzeletski determined that the Orgia [οργυιά?], near or equal to Attic unit, was a unit of measurement by means of which were delimited the allotments of Chersonesos in the Heraclean peninsula.[11] A.H. Shcheglov, having studied monuments of the Chersonesan chora [χωρα?] in the North-West Crimea, has arrived at an analogous conclusion.[12]

For standards of capacity of the centres aforementioned, up to this time there has not been devoted a special investigation; B.N. Grakov, who began to work on them and measured the capacities of 150 amphoras, did not succeed in completing this task. I.B. Zeest and V.V. Borisova present capacities of a few types of amphoras without defining their standards.[13] Recently in connection with an examination of types of amphoras V.I. Kats and S.Y. Monachov touched upon the question of the standard of capacity of amphoras.[14] Determination of standard sizes of capacity may throw light on many aspects of the economic life of an ancient town, about which I.B. Brashinsky speaks in some detail (op. cit. pp. 87-89).

144 In the present article the author sets forth the results of her work on the determination of standards of capacity of Chersonesan amphoras of the Hellenistic period, which she has endeavoured to establish with an application of a mathematical method of the calculation of the capacity of amphoras. Since the method of determining standards of capacity combines in itself linear and actual measurement of capacity, it is necessary to examine in greater detail the most prevalent systems of linear measures. It is known that, as the basis for the formation of measures of capacity and weight, linear measures served,[15] for which one cubic foot corresponds to the metrical expression of weight 'talent', one and a half feet to the weight of a metretes, two feet to the weight of a medimnos.

In most common use was a foot at 295.7-296 mm, being formed from the length of the Persian elbow [i.e. cubit?] at 443.55-444 mm and composing 4/5 of the Babylonian large cubit at 555 mm, and foot at 297 mm from the cubit at 445.5 mm, being formed from the old Babylonian cubit at 495 mm.[16] In literature this foot, equal to 296-297 mm, often is called Attic.

Also known is a foot at 326-328 mm from a cubit at 495mm (ibid.p.857). Dörpfeld called it Aeginetan and considered it the most generally used at Athens.[17] In works of recent years, to the foot of 296 mm has been applied the name of short, or Ionic, and to the foot of 326 mm the name long, Doric,

(144) or Attic.[18] In connection with these it is necessary to observe that researchers, speaking about the use at Chersonesos of the Euboic system of linear measures, had in mind a foot at 296mm, that is, Ionic.

For the bringing to light of standards of capacity, we measured 45 amphoras of Chersonesan manufacture which are preserved in storage places of Chersonesos and in other museums, and in addition we used the data on capacities presented in the work of V.V. Borisov.[19] The capacity of intact and restored amphoras was determined empirically, for which each amphora was filled with water or grain up to the crown ['halo' - I take this to mean lip] of the neck. The capacity of all amphoras was computed also by mathematical means.

The first experiment of mathematical calculation of the capacity of a container was performed by us on an example of a Sinopean pithos with numerical marks.[20] The full volume of the pithos was calculated on the formula known to mathematics:

$$V = \frac{\pi}{3}\sum (R_j^2 + R_j \cdot R_j + R_j^2)\Delta h$$

by the approximated calculation of the volumes of bodies of revolution. The meaning of this formula consists in this, that a body of revolution in relation to its axis is broken down into a series of elementary bodies of revolution, in the present case truncated cones. The sum of the volumes of these truncated cones gives the full volume of the body of revolution. The capacity of the pithos, calculated on this formula, comprised 1065 ± 65 l. ~~The difference~~ If the capacity is calculated according to the formula of Geron Hiero for pithoidal bodies, employed by M. Lang [21] for the calculation
145 of the volume of an hypothetical Thasian pithos, then it will comprise 984.25 l. The difference in the calculations it is possible to explain by this, that in the use of a comtemporary formula there arise errors in the tracing of the profile of the vessel, which depended on the practical deviation of the form of the pithos from an ideal one. These errors make up roughly 5% of the volume of the vessel. To determine errors in calculations on the formula of Hiero, which was obtained most probably by empirical means, is possible if one has at one's disposal much statistical material, which at the present time we do not yet have. Furthermore, M. Lang considers that one of the diametres, given in the formula of Hiero, is the inner diametre of the body. But this diametre is difficult to measure in practice. It was far more convenient for the master to control his wares

(145) by measuring the external diametre of the body and the internal diametre of the mouth. Just so the master proceeded when he was checking the dimensions of an amphora.[22] If, confirming our reasoning, we calculate the capacity of a pithos by substituting in the formula of Hiero the quantity of the external diametre of the body, then the capacity will be 1027.5 l. This result is close enough to the significance of the volume obtained in the formula of the sum of volumes of truncated cones and corresponds to 20 medimnoi or 26 metretes or 30 Attic cubic feet or 40 Ionic cubic feet.

Of this formula(the sum of truncated cones) we availed ourselves in the calculation of the capacity of amphoras. In the result of our meaurements it is possible to distinguish the standards of amphoras, the capacity of which fluctuates to the following limits:

End IV to mid III c. BC[23]

1st 31.5-32.5 l.; 2nd 19.40-19.60 l.; 3rd 17.20-17.50 l.; 4th 14.50-14.70 l.; 5th 4.80-5.00 l. Provisionally there are distinguished the standards: 6th 30.20-30.30 l.; 7th 27.20-27.50 l.; 8th 22.50-22.70 l.; 9th 9.60-9.80 l.

Mid III to II c. BC

8th 22.50-22,7o l.; 3rd 17.20-17.50 l.; 9th 9.60-9.80 l.; 5th 4.80-5.00 l. Provisionally there are distinguished the standards: 2nd 19.40-19.60 l.; and 10th 8.50-8.60 l.

Standards 1, 2, 4, 9 and 5 correspondingly equal 10, 6, 4, 5, 1.5 Euboic choes at 3.2 litres, or 11.5, 7, 5, 3.5, 1 2/3 Chian choes at 2.8 litres. Standards 3 and 10 equal 8 and 16 choeniks at 1.08 litres or 1/3 and 1/6 Euboic medimnoi, and also 6 and 3 Chian choes. The standards which are provisionally determined equal: 6th standard - 1Pontic maris [μαρις ?] at 30.31 litres; 7th - 8.5 Euboean choes or 9 1/3 Chian choes, 6 Pontic addiks or Aeginetan choes at 4.55 litres; 8th - 22.75 litres, i.e. 7 Euboean or 8 Chian choes, and also 5 Pontic addiks. The 4th standard equals also 1 Pontic hydria at 14.58 l., while standards 5 and 9 are respectively 1/3 and 2/3 of a Pontic hydria.

Probably the most wide-spread standard at the beginning of the IV to the mid III c, BC was the 2nd standard, 19.40-19.60 l., since amphoras of such capacity form 50% of the general quantity of amphoras of the time mentioned. For the mid III to II c. BC the most characteristic standard was, probably, the 5th, 4.80-5.00 l. (50% of the general number of amphoras of that time). The 3rd standard, 17.20-17.50 l. forms 25% of the general number of amphoras in the IV to II c. BC and, possibly, was wide-spread for

(145) the extent of that period.

Since all the standards contain a definite number of Euboean choes, it is probable the supposition that for the manufacture of an amphora the master made use of the Euboean standards of capacity, at the same time being of a sort which it is easy to recalculate into the standards of other systems.

On the foundation of the calculations carried out,, it is possible to state a hypothesis about it that in Chersonesos simultaneously there existed standard fractional vessels, and also, that the basic standard was different in different periods. This supposition rests on the conclusions of V. Grace, who on the basis of treatment of a substantial quantity of material arrived at a conclusion about the standardness of sharp-bottomed amphoras, about the existence of several standards in a single centre
146 simultaneously, and about the alteration of standard throughout the extent
of a definite segment of time in one centre.[24] The diversity of standards and the sizable capacity of an amphora of the basic standard (19.40-19.60) probably is explained by the fact that Chersonesos at the end IV to beginning III c. BC produced a significant quantity of wine for sale. Decrease of the standard of capacity in the III to II c. BC probably was conditioned by the fact that as a result of wars with the Scythians the city loses a considerable part of her territory, and in consequence of this production of wine decreased.

It is known that Chersonesos in her out-put of a container imitated the types of Pontic Heraclea and Sinope.[25] N.A. Leipunskaya observed the monotony of amphoras of Heraclean manufacture within the type, which was caused, in the opinion of the author, by the standardized character of the ceramic production and trade of Heraclea.[26] The basic linear parametres of amphoras of that centre vary within the limits:ht. of vessel 61-75.5 cm, d. of body 22-29 cm, d. of mouth 8-10 cm.[27] The linear measurements of amphoras of Chersonesan production, which imitate the shapes of Heraclean ones, are confined to those same limits. One should notice also that a considerable quantity of Chersonesan amphoras are near to the Sinopean ~~xxxxxxxxxxx~~, Thasian and Pantikapaean measurements.

The production of the ceramic container was carried out probably in accordance with indicated state standards,[28] and the coincidence of dimensions of amphoras of Chersonesan manufacture with amphoras of the enumerated centres probably testifies that Chersonesos, which was a sizable

(146) trading centre, aimed at making uniform the standards of measures, which simplified the accomplishment of commercial operations. By investigators an important fact has been observed: measurements of amphoras of different types give a definite firm monotony of the metric features in amphoras of a single type, namely: height, depth, diametre of the body, of the mouth, height of the upper part (up to the line of the diametre of the body), correlation of the height of the lower part of the amphora and its full height.[29]

Work on the determination of proportional dependance between different parts of an amphora was undertaken by N.A. Leipunskaya. She calculated the proportional dependance between the height of the lower part of the amphora and its full height. The ideal quotient of this proportion was 0.618,[30] that is, 3/5 of the height of the amphora. Analogous computations conducted on Chersonesan amphoras confirms this result. Hence, the ratio of the height of the upper part of the amphora to its whole height in amphoras of Chersonesan manufacture fluctuates at the limits 0.200-0.247, that is 1/5 - 1/4 of the height of the amphora, and the ratio of the diametre of the neck to the diametre of the body is o.210-0.240.

According to their linear parametres, Chersonesan amphoras of definite capacities can be distributed into several groups: 1st cap. 5 l., ht. 50-54 cm, d. of the body 21-24 cm; 2nd cap. 10-11 l., ht. 68-70 cm, d. body 24-28 cm; 3rd cap. 14-15 l, ht 69-71 cm, d. body 27-28 cm, 4th cap. 16.5-19.5 l, ht 70-71 cm, d. body 29-30 cm; 5th cap. 22-27 l, ht 70-71 cm, d. body 30-36 cm; 6th cap. 30-32 l, ht 69-72 cm, d. body 35.5-37 cm; 7th cap. 22-27 l, ht 75-88 cm, d. body 53-57 cm.

The height of amphoras of the 1st group makes up ¾ of the height of the amphoras of the 2nd-6th groups, the diametres of the body of amphoras of the 1st-6th groups make up respectively 1/3, 2/5, 2/5, 2/5, 1/2, 1/2 of the height of the amphoras of the 2nd-6th groups. The diametre of the neck of all groups of amphoras is 8-10 cm, which makes up 1/9 of the height of
147 amphoras of the 2nd-6th groups. The depth of amphoras of the 1st group constitutes 3/5 of the height of the 2nd-6th groups, the depth of which is 9/10 their height.

Proceeding from this table, it is possible to make up a series of correlations of heights and diametres of amphoras of the 1st to 6th groups, where a height of 69-71 cm, has been accepted as a unit, that is the

(147) height of amphoras of the greatest capacity, and the most frequently repeated height(in all groups except the 1st). The series looks thus: 1 : 1/9 : 1/5 : 2/5 : 3/5 : 1/3 : 1/2 : 3/4 : 9/10.

Finally, to the master these correlations were well-known, and for the production of an amphora of determined capacity it was enough for him to have one measuring instrument, on which all the necessary dimensions were accumulated.

The master produced an amphora of a set capacity using a template.[31] For this, in order to manufacture the template, it is necessary to calculate a theoretical capacity of the vessel. About the formulas of Hiero for computation of a theoretical capacity of pithoi of different shape it has already been spoken. For the computation of the capacity of amphoras such formulas are not known. But, as the investigations of I.B. Brashinsky have shown, for the computation of the capacity of amphoras of pithoid type it is possible to employ the above-cited formula of Hiero.[32] The use of these formulas of Hiero for the computation of capacities of all types of sharp-bottomed amphoras M. Lang and I.B. Brashinsky consider inaccurate.[33] However the introduction of a coefficient into the formula of Hiero makes it, in our view, acceptable for this purpose. We shall explain the expressed proposition.

Chersonesan amphoras, as also amphoras of many other centres, it is possible to break down into geometrical figures: cylinder (neck), truncated cone (intervening space between line of the base of the neck and the line of the maximum diametre) and cone (lower portion of the amphora from the line of the maximum diametre). To obtain the capacity of an amphora is possible by making up the sum of the volumes of these figures, calculated according to the adopted formula:

$$V = \left(\frac{1}{4}\pi d^2 \text{ cylinder} \times h \text{ cylinder}\right) + \left(\frac{1}{12}\pi h \text{ truncated cone} \times (d^2 \text{ cylinder} + d \text{ cylinder} \times d \text{ body})\right) + \left(\frac{1}{12}\pi d^3 \text{ body} \times h \text{ cone}\right)$$

If we are to make use of the calculations of the correlation of the height of an amphora with the height of the separate parts, and of the diametre of the body with the diametre of the neck, and to express in the

(147) given formula the diametre of the neck and the heights of the separate parts of the amphora through these correlations, the formula will appear:

$$V = \left(\frac{1}{4}\pi \times 0.24d^2\text{body} \times 0.25h \text{ amphora}\right) + \left(\frac{1}{12}\pi \times 0.13h \text{ amphora} \times (0.24d^2\text{body} + 0.24d \text{ body} \times d\text{body} + d^2\text{body})\right) + \left(\frac{1}{12}\pi d^2\text{body} \times 0.62h \text{ amphora}\right)$$

After calculations we obtain the result $V = 0.259d^2h$

Now taking up the formula of Heiro: $V = \frac{11}{14} \times \frac{(d \text{ neck} + d \text{ body})^2}{2} \times h$

and conducting the same operation. The final result will look thus: $V = 0.243d^2h$. [Translator's note: All these calculations use the substitution of 0.24dbody for dneck, but in none of them does the author appear to square the whole term where dneck should have been sqared. She uses $0.24d^2$body where she should have $0.058d^2$body.] Comparing the results of calculations in accordance with these formulas, it is possible to say that they are

[sic. R.T.B.] approximatedly equal. This gives the basis for utilizing Heiro's formula for calculations of the capacity of sharp-bottomed amphoras.

Rearranging the formulas, we substituted the maximal value of the correlations of diametres and heights. If we substitute in these formulas the minimal value of the correlations, then in the final formula we will obtain a new value of the coefficient. Hence it is possible to draw the conclusion that the value of the coefficient depends on the correlations of the diametres and heights of the different parts of the amphora, in other words, on its proportions. In the last formula this will look thus: $V = xd^2h$.

For amphoras of Chersonesan manufacture the value of x for each type of amphoras has been determined according to the formula $x = \frac{V}{d^2h}$. Since the value of the coefficient was obtained on not numerous material, to maintain its stability for Chersonesan as also for amphoras of other centres of production is for the time being premature. Nevertheless, endeavouring to check the results of our xkx calculations on amphoras of Pontic Heraclea and Sinope, we obtained a satisfactory result: the capacity of an amphora

148 calculating empirically nearly coincides with the capacity calculated on the given fromula.[34] A difference of 200-300 gm practically can be disregarded.

(148) We do not know how the formula looked for ~~the~~ calculation of bodies of revolution before Hiero, but we may suppose that the Greeks, in as much as they knew ~~Egyptian~~ the mathematics of the Egyptians,[35] used their formulas. It is known that the formula of the volume of a cylinder among the Egyptians was $\left(\frac{8}{9}\right)^2 D^2H$, in modern interpretation $\frac{\pi}{4} D^2H$. In order to calculate the volume of a body of revolution of another form, the Egyptians introduce to the formula of the volume of a cylinder a coefficient.[36] Relying on this knowledge, it is possible to say that the formula obtained by us for the calculations of volumes pertains to bodies (in the present instance amphoras) of such a form as requires the introduction of a coefficient corresponding to the formula of the volume of a cylinder. In other words, our formula may appear: $V = x\frac{\pi}{4}D^2H$.

Analysing the material obtained, it is possible to draw the following conclusions:

1. In Chersonesos in the Hellenistic period ~~thate~~ there are distinguished definite standards of measures of capacity, among which for the end IV to mid III c. BC the standard 19.40-19.60 l. is the most characteristic, and for the mid III to II c. BC the standard 4.80 to 5.00 l. The fluctuation of the standards is accounted for, probably, by a change of the condition of the economic basis of Chersonesos.[37]

2. In this same period in Chersonesos there is to be observed a definite unification of standards of measures. On the face of it the standards apparently are diverse (in building and architecture there has been established the use of the Ionic foot, in trade, of the Chian-Rhodian and Persian monetary-weight systems, in ceramic prodution, of the Euboic and, probably, of the Pontic standards of capacity). However the weight of capacity, filled with water, which is equal to the weight of the Chian-Rhodian drachma (3.88 gm), makes up 1/10,000 of the Persian drachma, 1/7,000 of the Euboic metretes (of 38.88 kg), the metric equivalent of which is 1.5 cubic feet at 296 mm. In other words, standards of capacity and of weight it is possible to reduce to a common module: the Ionic foot of 296 mm.

3. Comparing the linear parametres of amphoras of various centres, it is possible to observe a striving towards standardization of linear

(148) measures of this type of production in specific regions, in particular in Chersonesos and centres which trade with her: Heraclea Pontica, Sinope, Thasos, Pantikapaion.

4. Linear Dimensions of amphoras are found in proportional dependance which is determined by the shape of the amphora. In a formula this dependance is expressed by a coefficient.

5. For mathematical calculation of the capacity of sharp-bottomed amphoras it is possible to apply the formula $x\left(\frac{8}{9}\right)^2 D^2 H$, using for each type of amphoras a specific coefficient. Such calculations give the possiblility of determining the capacity of fragmented vessels, which is particularly important for the analysis of ceramic material, the principal quantity of which consists of fragments of amphoras.

6. Probably not only standards of measures but also methods of calculation of them the Greeks strove to make uniform, which had great significance in ancient manufacture and trade.

Footnotes

1) H. Nissen, Griechische und romische Metrologie, Handbuch der klassischen Altertumswissenschaft, I[2] Munchen, 1892, p. 850 f.

2) I. B. Brashinscky, Methods of study of standards of ancient Greek ceramic containers, SA 1976, 3, p.87-104.

3) A. L. Bertye-Delagard, Relative value of monetary metals in the Bosporos and Boristhenos in the middle of the IV c. BC. Numismatic collection, vol. 1, Moscow, 1911; A.N. Zograph, Ancient coins, M.I.A., 16, 1951; D.B. Shelov, Monetary affairs of the Bosporos IV-II c. BC, Moscow, 1956

4) Zograph, op. cit. p. 125, 127.

5) Shelov, op. cit., p. 73, 83.

6) L. I. Chuistova, Ancient and medieval weight systems circulating in the north Black Sea area, in the collection Archaeology and history of the Bosporos, vol. 2, Simpheropol 1962.

7) N. L. Grach, Lead weights from Nymphea and some questions of Bosporic weight metrology, Works of the State Hermitage, vol. XVII, Leningrad, 1976, p. 194.

8) E. O. Berzin, On linear measures of the Bosporos, SA XXVI, 1956, p. 227-235.

9) N. Bolotin, Antike Masse in ihrer Bedeutung fur Geschichte und Kunst, Das Altertum, Bd. 15. 1956, Ht. 4, p. 216 f.

10) I.B. Brashinski, Standards of linear measures in the ceramic production of Sinope IKAM, Moscow, 1977, p. 35 f.

11) S. F. Strzheletski, Holdings of Chersonesos in Tauros, X.Sb. [Xersoneski Sbornik = Chersonesan Collection], VI, Simpheropol, 1961, p. 56-57.

12) A. N. Shcheglov, The North-West Crimea in ancient times. Synopsis of thesis for a master's degree, Leningrad, 1971, p. 14; idem, Dwelling place of Hellenistic Kalos Limena, Artistic Culture and Archaeology of the Ancient World, Moscow, 1976, p.235.

13) I.B. Zeest, Ceramic container of the Bosporos M.I.A., 83, 1960, p. 75 f.; V.V. Borisova, Ceramic stamps of Chersonesos and classifications of Chersonesan amphoras, N.E., XI, 1974, p. 102, 111.

14) V.I. Kats, S.Y. Monakhov, Amphoras of Hellenistic Chersonesos, Ancient World and Archaeology, extract III, Saratov, 1977, p. 102 f. Agreeing with the conclusions of the authors about standards of capacity, calculated by them by empirical means, it is neceessary to observe that in the quantity of amphoras used for analysis (10 examples), it is impossible to construct a histogram of the capacities. For the obtaining of the most

Footnotes (contd)

14) (contd) probable conclusion by a histogram, the number of statistical similar data (observations) ought to approach closer to a hundred (cf. E.S. Venttsel, The Theory of Probability, Moscow, 1962, p.192)

15) In more detail about this cf. Nissen, op. cit. p. 849; N.T. Belyaev, About ancient and modern Russian measures of length and weight, Seminarium Kondakovianum, vol. I, Prague 1927, p. 247 f.

16) Nissen op. cit. p. 861, 857.

17) W. Dörpfeld, Metrologische Beitrage, Mittheilungen des Deutschen Archäologischen Instituts in Athen, XV, 1890, p. 167.

18) M. Lang, Anew inscription from Thasos: Specifications for a measure, BCH, 76, 1952, p. 22-28; W. Dinsmoor, The Basis of Greek Temple Design: Asia Minor, Greece, Italy, Atti del settimo Congresso Internationale di archeologia classica, vol. I, Roma, 1961, p. 358 f. In Lang the values of feet are 296 and 326 mm, in Dinsmoor 293.9 and 326.55 mm. In our subsequent calculations we will use the Ionic foot of 296 mm and Doric foot of 326 mm.

19) Borisova, op. cit. p. 102-104. Measurements of amphoras from excavations Panskoe I conducted by S.Y. Monakhov.

20) G.M. Nikolaenko, Marks on ancient pitoi, Tauric Chersonesos, Craftsmenship and Culture, Kiev, 1974, p. 29.

21) Lang, op. cit., p. 28. The formula appears:

$$V = \frac{11}{24}\left(\frac{D_{mouth} + D_{body}}{2}\right)^2 \times h,$$

where h, in the opinion of M. Lang, is the depth of the pithos. There is known also the formula of Hiero for the calculation of the capacity of a spheroid pithos:

$$V = \frac{11}{21}\left(\frac{D_{mouth} + D_{body}}{2}\right)^2 \times h.$$

About this see Metrologicum scriptorum reliquiae, (ed. Fr. Hultsch), Lipsiae, 1864, p. 202 f. No. 19-20.

22) Brashinski, op. cit., p. 97, note.

23) Date of ceramic material given in mentioned work of V.V. Borisova.

24) VG, Standard pottery containers....p. 175.

25) Borisova, op. cit., p. 101.

26) N.A. Leypunskaya, About standard Heraklean amphora containers, Olvia, Kiev, 1975, p. 130.

27) Ibid., p. 127.

28) Brashinski, Methods of investigation...,p. 90.

29) Ibid., p. 93; Leypunskaya, op. cit., p. 131.

Footnotes (contd)

30) N.O. Leipunskaya, The principle of proportionality in ancient ceramic ? 'virobs' (in Ukrainian), Archaeologiya, 15, 1975, p. 25 f.

31) Leypunskaya, About standards ;..., p. 133; Brashinski, Methods of investigation..., p. 90 f.

33) Ibid., p. 92-93.

34) The arithmetical value of the coefficient I do not give, since it was obtained on an insufficient quantity of material.

35) M. Y. Vigodski, Arithmetic and algebra in the ancient world, Moscow, 1967, p. 9.

36) G.P.Boev, Calculation of surfaces and volumes of bodies of revolution among the ancient Egyptians, VDI, 1950, no. 3, p. 200.

37) About this change political events can testify: the loss of holdings in the North-West Crimea toward the beginning of the II c. BC, raids of barbarians on the urban region of Chersonesos in the III to II c. BC, destruction of country settlements of potters and wine-makers.

NIKOLAENKO

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Reference
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[p142]

On Standards of Capacity of Hellenistic Khersonesos

Thorough study of material on the economy and trade of ancient states requires more and more active application of data on metrology. The importance of metrological investigations for the study of these problems was already noted in the last century by H. Nissen.<f1> Unfortunately, little attention has been paid to this theme until now. A work of I.B. Brashinskii which has recently appeared has to do with questions of the methods of determining the capacities of ceramic containers.<f2> Since the author characterizes sufficiently fully the state reached in studying this problem, we will not dwell on it in detail. Let us remark only that the basic attention of the investigators was concentrated on the study of the weight and linear metrology of Olbia, Pantikapaia and Khersonesos, predominately the weight standards.

=== 1. H. Nissen, <i>Griechische und r\"omische Metrologie, Handbuch der klassischen Altertumswissenschaft<i>, I^{2}, M\"unchen, 1892, p850 f.
=== 2. I.B. Brashinskii, "Methods of Studying the Standards of Ancient Greek Ceramic Containers [Metodika izucheniia standartov drevnegrecheskoi keramicheskoi tary]," <i>SA<i> 1976, nr 3, pp87-104.

[p143]
The first metrological researches on the north Black Sea area weight system were undertaken by A.L. Berte-Delagard, whose conclusions were later amplified by A.N. Zograf and D.B. Shelov.<f3> Among the prominent monetary-weight systems recorded in the north Black Sea area are the Rhodian-Chian, Persian, Aeginetan, Euboian, and others.<f4> The Aeginetan system was predominate at Olbia and Pantikapaia,<f5> the Rhodian-Chian at Khersonesos (Zograf, <i>op. cit.<i> p147). Here also the Persian system was deduced, while the Euboian system was deduced at Pantikapaia and Olbia. In the opinion of L.I. Chuistova, one may distinguish a group of weights on Khersonesos' own system as well with a mna weight of 207 gm.<f6> But since she proceeds on an incorrect reading of the letter marking on a stone weight (39 instead of 19), this conclusion cannot be recognized as successful. Furthermore, the method of weighing the metal weights which L.I. Chuistova adopted was incorrect,<f7> and the conclusions of the author cannot therefore be used.

=== 3. A. L. Berte-Delagard, "The relative value of monetary metals in the Bosphoros and Borisphen in the mid-IV century BCE. [Otnositel'naia stoimost' monetnykh metallov v Bospore i Borisfene v polovine IV v. do n. e.]," <i>Numizmaticheskii sbornik<i>, vol. I Moscow 1911; A.N. Zograf, "Ancient coinage [Antichnye monety], <i>MIA<i> 16 1951; D.B. Shelov, <i>Coin production in the Bosphoros IV-III centuries BCE [Monetnoe delo Bospora IV-III vv. do n. e.], Moscow 1956.

=== 4. Zograf, <i>op. cit.<i> p125, 127.

=== 5. Shelov, <i>op. cit.<i> p73, 83.

=== 6. L.I. Chuistova, <i>Ancient and mediaeval weight systems, circulating in the north Black Sea area [Antichnye i srednevekovye sistemy, imevshie khozhdenie v Severnom Prichernomor'e]<i>, in coll. "Archaeology and History of the Bosphoros [Arkheologiia i istoriia Bospora]," vol. II, Simferopol' 1962.

=== 7. N.L. Grach, <i>Lead weights from Nymphea and some questions of Bosporan weight metrology [Svintsovye giri iz Nimfeia i nekotorye voprosy bosporskoi vesovoi metrologii]<i>, "Works of the State Hermitage [Trudy Gos. Ermitazha]," vol. XVII, Leningrad 1976, p194.

In characterizing linear standards of the Bosphoros, E.O. Berzin notes that here, as in Olbia, the Attica-Euboian (Ionian) system was used.<f8> N.I. Bolotin also isolates the Euboian (Ionian) system in Pantikapaia, but in Olbia he considers the Aeginetan to be the basic one;<f9> however, I.B. Brashinskii observed that the roof-tiles of this centre as well as of Khersonesos were made according to the Attic (foot at 326 mm) standard.<f10> The question of the linear measures of Khersonesos has been partially developed. S.F. Strzheletskii determined that the unit of measurement with the aid of which the allotments of Khersonesos on the Heraclean peninsula were divided was the orguia, close to or equal to the Attic orguia.<f11> Studying the monuments of the khora of Khersonesos in the north-western Crimea, A.N. Shcheglov arrived at an analogous conclusion.<f12>

=== 8. E.O. Berzin, "On Linear Measures of the Bosphoros [O lineinykh merakh Bospora]," <i>SA<i> XXVI 1956 pp227-235.

=== 9 N. Bolotin, "Antike Masse in ihrer Bedeutung f\"ur Geschichte und Kunst," <i>Das Altertum<i> Bd. 15 1959 Ht. 4 p216 f.

=== 10. I.B. Brashinskii, "Standards of Linear Measures in Ceramic Production at Sinope [Standarty lineinykh mer v keramicheskom proizvodstve Sinopy]," <i>IKAM<i> Moscow 1977 p35 ff.

=== 11. S.F. Strzheletskii, "Kleroi of the Tavridean Khersonesos [Klery Khersonesa Tavricheskogo]," <i>KhSb<i> VI Simferopol' 1961 pp56-57.

=== 12. A.N. Shcheglov, <i>The North-western Crimea in Ancient Times [Severo-Zapadnyi Krym v antichnuiu epokhu]<i>. Abstract of PhD diss. Leningrad 1971 p14; <i>idem<i>, "Residential Housing in Hellenistic Kalos Limen [Zhiloi dom ellinisticheskogo Kalos Limena]," <i>Artistic Culture and Archaeology of the Ancient World [Khudozhestvennaia kul'tura i arkheologiia antichnogo mira]<i>, Moscow 1976 p235.

So far there has been no special study devoted to the standards of capacity in the above-mentioned centres; B.N. Grakov, who began this work and measured the capacities of 150 amphoras, did not manage to complete the work. I.B. Zeest and V.V. Borisova give capacities of some types of amphoras without determining their standards.<f13> Recently in connection with studying the types of amphoras, V.I. Kats and S.Iu. Monakhov touched upon the question of the standards of amphora capacities.<f14> Determination of standard sizes of capacity may shed light on many aspects of the economic life in an ancient city as I.B. Brashinskii discusses in detail (<i>op. cit.<i> pp87--89).

=== 13. I.B. Zeest, "Ceramic Containers of the Bosphoros [Keramicheskaia tara Bospora," <i>MIA<i> 83 1960 p75 ff; V.V. Borisova, "Ceramic Stamps of Khersonesos and the Classification of Khersonesan Amphoras [Keramicheskie kleima Khersonesa i klassifikatsiia khersonesskikh amfor]," <i>NE<i> XI 1974 p102, 111.

=== 14. V.I. Kats, S.Iu. Monakhov, "Amphoras of Hellenistic Khersonesos [Amfory ellinisticheskogo Khersonesa]," <i>The Ancient World and Archaeology [Antichnyi mir i arkheologiia]<i> III Saratov 1977 p102 ff. While agreeing with the authors' conclusions on the standards of capacity calculated by them empirically, one must observe that it is not permissible to build a histogram of capacities based on the quantity of amphoras (10 specimens) used for the analysis. In order to obtain the most probable conclusion by histogram the number of statistical homogeneous data (observations) must approach 100. (see E.S. Venttsel', <i>The Theory of Probability [Teoriia veroiatnosti]<i> Moscow 1962 p192.)

[p144]

In the present article the author presents the results of work on determining the standards of capacity of Khersonesan amphoras of the Hellenistic period which she attempted to establish by applying a mathematical method of calculating amphora capacities. Since the methodology of determining standards of capacity combines linear and factual measurements of capacities it is necessary to examine the most wide-spread systems of linear measures in greater detail. As is known, the linear measures were the basis for forming the measures of capacity and weight,<f15> where one cubic foot corresponded to the metric expression of the weight of a talant, one and a half feet to the weight of a metretes, and two feet to the weight of a medimnus.

=== 15. For more detail see Nissen <i>op. cit.<i> p849; N.T. Beliaev, "On Ancient and Present Russian Measures of Distance and Weight [O drevnikh i nyneshnikh russkikh merakh protiazheniia i vesa]", <i>Seminarium Kondakovianum<i> I Prague 1927 p247 f.

The most widely used were a foot of 295.7-296 mm, formed from the length of a Persian or Phoenician cubit of 443.55-444 mm and comprising 4/5 of a Babylonian large cubit of 555 mm, and a foot of 297 mm from a cubit of 445.5 mm formed from the ancient Babylonian cubit of 495 mm.<f16> In literature this foot equal to 296-297 mm is often called the Attic foot. A foot of 326-328 mm from a cubit of 495 mm is also known (<i>ibid.<i> p857). D\"orpfeld called it the Aeginetan foot, and considered it to be the most widely used in Athens.<f17> In the works of the last few years, a foot of 296 mm has acquired the name of short or Ionian, and a foot of 326 mm long, Dorian, or Attic.<f18> In connection with this it is necessary to note that researchers speaking of the use of the Euboian system of linear measures in Khersonesos had in mind a foot of 296 mm, i.e., Ionian.

=== 16. Nissen <i>op. cit<i> pp861, 857.
=== 17. W. D\"orpfeld, "Metrologische Beitr\"age," <i>AthMitt<i> XV 1890 p167.
=== 18. M. Lang, "A New Inscription from Thasos: Specifications for a Measure," <i>BCH<i> 76 1952 pp22-28; W. Dinsmoor, "The Basis of Greek Temple Design: Asia Minor, Greece, Italy," <i>Atti del settimo Congresso Internationale di archeologia classica<i>, vol. I, Rome 1961 p358 f. Lang has foot values of 296 and 326 mm and W. Dinsmoor 293.9 and 326.55 mm. In our further

calculations we will use the Ionian foot of 296 mm and Dorian foot of 326 mm.

To determine the standards of capacity we measured 45 amphoras of Khersonesan production preserved in the collections of the Khersonesan and other museums, and moreover we used the data on capacities given in the work of V.V. Borisova.<f19> The capacity of whole and restored amphoras was determined empirically whereby each amphora was filled with water or grain to the rim of the neck. The capacity of all amphoras was calculated mathematically as well.

=== 19. Borisova <i>op. cit.<i> pp102-104. The measurements of amphoras from the Panskoe-I excavations were done by S.Iu. Monakhov.

The first experiment of mathematical calculations of container capacity was carried out by us on the example of a Sinopean pithos with numerical markings.<f20> The full volume of the pithos was calculated by the well-known mathematical formula $V = \pi/3 \times \sum(R_{j}^{2} + R_{j}.R_{j} + R_{j}^{2}) \times \Delta h$ for approximate calculation of the volumes of bodies of rotation. The meaning of this formula consists in the fact that the body of rotation is broken along the axis into a series of elementary bodies of rotation, in this case truncated cones. The sum of the volumes of these truncated cones gives the full volume of the body of rotation. The capacity of the pithos calculated by this formula comprised 1065 +/- 65 litres. If we calculate the capacity by Heron's formula for pithoid bodies used by M. Lang<f21> for calculating the volume of a hypothetical Thasian pithos, it will comprise [p145] 985.25 litres. The difference in calculations may be explained by the fact that in using a contemporary formula there appear inaccuracies in drawing the profile of the vessel which is caused by practical deviations of the pithos form from the ideal. These inaccuracies comprise approximately 5% of the vessel's volume. One can determine the inaccuracies in calculations by Heron's formula, arrived at most likely in an empirical way, by the availability of a large body of statistical material which we don't have at present. Moreover M. Lang considers that one of the diameters given in Heron's formula is the internal diameter of the body. But this diameter is hard to measure in practice. It was much more convenient for a master to check his artefact by measuring the external diameter of the body and the internal diameter of the mouth. Undoubtedly the master acted in the same way when he checked the size of the amphora.<f22> If in order to confirm our reasoning we calculate the capacity of a pithos by putting the value of the external diameter of the body into Heron's formula, it will comprise 1027.5 litres. This result is quite close to the value of the volume arrived at by the formula of the sum of the volumes of the truncated cones and corresponds to 20 medimni or 26 metretes or 30 Attic cubic feet or 40 Ionic cubic feet.

=== 20. G.M. Nikolaenko, "Markings on Ancient Pithoi [Metki na antichnykh pifosakh]," <i>Tavridean Khersonesos. Crafts and Culture [Khersones Tavricheskii. Remeslo i kul'tura]<i>, Kiev 1974 p29.

=== 21. Lang <i>op. cit.<i> p28. The formula looks like this: V = 11/14 x ((Dmouth + Dbody)/2)^{2} x h, where h in the opinion of M. Lang, is the depth of the pithos. A formula of Heron for calculating the capacity of a spheroid pithos is also known: V = 11/21 x ((Dmouth + Dbody)/2)^{2} x h. On this see <i>Metrologicum scriptorum reliquiae<i> (ed. Fr. Hultsch), Lipsiae 1864 p202 ff. nr 19-20.

=== 22. Brashinskii <i>op. cit.<i> p97 note.

We used the same formula (the sum of truncated cones) in calculating the capacity of amphoras. As a result of measurements one can isolate standards of amphoras, the capacity of which fluctuates within the following limits:

End of IV--mid III BCE<f23>

<i>First<i>: 31.5--32.5 l.; <i>second<i>: 19.40--19.60 l.; <i>third<i>: 17.20--17.50 l.; <i>fourth<i>: 14.50--14.70 l.; <i>fifth<i>: 4.80--5.00 l. Conjecturally we isolate standards: <i>sixth<i>: 30.20--30.30 l.; <i>seventh<i>: 27.20--27.50 l.; <i>eighth<i>: 22.50--22.70 l.; <i>ninth<i>: 9.60--9.80 l.

Mid III--II BCE

<i>eighth<i>: 22.50--22.70 l.; <i>third<i>: 17.20--17.50 l.; <i>ninth<i>: 9.60--9.80 l.; <i>fifth<i>: 4.80--5.00 l. Conjecturally we isolate standards: <i>second<i>: 19.40--19.60 l., and <i>tenth<i>: 8.50--8.60 l.

=== 23. The dating of ceramic material is given according to the cited work of V.V. Borisova.

Standards 1, 2, 4, 9, and 5 are equal to 10; 6; 4; 5; 3; 1.5 Euboian choes of 3.2 l. each, or to 11.5; 7; 5; 3.5; 1 2/3 Chian choes of 2.8 l. each respectively. Standards 3 and 10 are equal to 8 and 16 choinikes of 1.08 l. each or 1/3 and 1/6 of a Euboian medimnus, as well as to 6 and 3 Chian choes. The standards determined conjecturally equal: sixth standard is equal to 1 Pontic maris of 30.31 l.; seventh to 8.5 Euboian chous [sic] or 9 1/3 Chian chous; to 6 Pontic addikes or an Aeginetan choe of 4.55 l.; eighth to 22.75 l., i.e., 7 Euboian or 8 Chian choes as well as to 5 Pontic addikes. The fourth standard is equal also to 1 Pontic hydria of 14.58 l. and the fifth and ninth to 1/3 and 2/3 Pontic hydrias respectively.

Probably the most wide-spread standard of the end of the IV--mid III BCE was the second standard, 19.40--19.60 l., since amphoras of such capacity constitute 50% of the total quantity of amphoras of the given period. For mid III--II BCE the most typical was probably the fifth standard, 4.80--5.00 l. (50% of the total number of amphoras of that time). The third standard, 17.20--17.50 l., constitutes 25% of the total number of amphoras of IV--II BCE, and was possibly wide-spread during this period.

Since all standards contain a definite number of Euboian choes it is possible to suppose that in making amphoras a master used

Euboian standards of capacity, such as may easily be converted into the standards of other systems.

On the basis of the given calculations one could put forward a hypothesis that in Khersonesos there existed simultaneously standard fractional vessels, as well as that the basic standard was different in different periods. This supposition arises from the conclusions of V. Grace who, on the basis of processing a considerable amount of material, came to the conclusion of the standard nature of pointed amphoras, of the existence of [p146] several standards simultaneously in one centre, and of the change of standard during a certain period of time in one centre.<f24> The variety of standards and considerable capacity of an amphora of the basic standard (19.40--19.60 l.) is possibly explained by the fact that at the end of the IV--beginning III BCE Khersonesos produced a considerable amount of wine for sale. The decrease of the standard of capacity in the III--II BCE is probably caused by the fact that as a result of wars with the Scythians the city lost a considerable part of its territory and consequently the production of wine was curtailed.

=== 24. V. Grace, "Standard Pottery Containers of the Ancient Greek World," <i>Hesperia<i> Supp. 8 1949 p175.

It is known that in the production of containers Khersonesos imitated the types of Pontic Heraclea and Sinope.<f25> N.A. Leipunskaia noted the homogeneity of the amphoras of Heraclean production within a type, which was caused in the author's opinion by the standardized character of the ceramic production and trade of Heraclea.<f26> Basic linear parameters of the amphoras of this centre fluctuate within the limits: height of vessel: 61--75.5 cm, diameter of body 22--29 cm, diameter of mouth 8--10 cm.<f27> Linear sizes of amphoras of Khersonesan production imitating Heraclean forms fall within these boundaries. It is also necessary to point out that a considerable number of Khersonesan amphoras are close to Sinopean, Thasian and Pantikapaian in their dimensions.

=== 25. Borisova <i>op. cit<i> p101.
=== 26. N.A. Leipunskaia, "On the standards of Heraclean amphora containers [O standartakh gerakleiskoi amphornoi tary]," <i>Olbia [Ol'viia]<i> Kiev 1975 p130.
=== 27. <i>ibid<i>. p127.

Production of ceramic containers was probably implemented according to state-defined standards,<f28> and the coincidence in the size of amphoras of Khersonesan production with amphoras of the above-mentioned centres possibly is witness to the fact that Khersonesos, which was a significant trade centre, strove to make the standards of measures uniform which simplified the conduct of trade operations. The scholars noted an important fact: measuring amphoras of different types produces a definite stable homogeneity of metric features in amphoras of the same type, that is height, depth, diameter of body, mouth, the height of the upper part (to the line of the diameter of the body), correlation between the height of the lower part of the amphora and its full height.<f29>

=== 28. Brashinskii, "Methods of Studying ..." p90.
=== 29. <i>Ibid.<i> p93; Leipunskaia <i>op cit.<i> p131.

The work on determining the proportional relation between the different parts of an amphora was carried out by N.A. Leipunskaia. She calculated the proportional relation between the height of the lower part of an amphora and its full height. The ideal quotient of this proportion is 0.618,<f30> i.e., 3/5 of the height of the amphora. Analogous calculations, carried out on Khersonesan amphoras, confirmed this result. Therefore the relation of the height of the upper part of an amphora to its full height is 0.368, i.e., 2/5 of the height of the amphora. Moreover it is established that the relation of the height of the neck to the full height of amphoras of Khersonesan production fluctuates within the limits of 0.200--0.247, i.e., 1/5--1/4 of the height of the amphora and the relation of the diameter of the neck to the diameter of the body is 0.210--0.240.

=== 30. N.O. Leipuns'ka, "The Principal of Proportionality in Ancient Ceramic Containers [in Ukranian]," <i>Arkheologyia<i> 15 1975 p25 ff.

According to the linear parameters Khersonesan amphoras of certain capacities may be divided into several groups: <i>first<i>: capacity 5 l., height 50--54 cm, diameter of body 21--24 cm; <i>second<i>: capacity 10--11 l., height 68--70 cm, diameter of body 24--28 cm; <i>third<i>: capacity 14--15 l., height 69--71 cm, diameter of body 27--28 cm; <i>fourth<i>: capacity 16.5--19.5 l., height 70--71 cm, diameter of body 29--30 cm; <i>fifth<i>: capacity 22--27 l., height 70--71 cm, diameter of body 30--36 cm; <i>sixth<i>: capacity 30--32 l., height 69--72 cm, diameter of body 35.5--37 cm; <i>seventh<i>: capacity 22--27 l., height 75--88 cm, diameter of body 53--57 cm.

The height of the amphoras of the first group comprises 3/4 of the height of the amphoras of the second to sixth groups. The diameters of the bodies of the amphoras of the first to sixth groups comprise 1/3, 2/5, 2/5, 2/5, 1/2, 1/2 of the heights of the amphoras of the second to the sixth groups respectively. The diameter of the neck of all groups of amphoras is 8--10 cm, which comprises 1/9 of the height of the amphoras of the second to sixth groups. The depth [p147] of the amphoras of the first group comprises 3/5 of the height of the second to the sixth groups, the depth of which is 9/10 of their height.

Based on this table one can compile a set of correlations between the heights and diameters of the amphoras of the first to sixth groups where the height of 69--71 cm, i.e., the height of the amphoras of the greatest capacity and most frequently repeated (in all groups except the first), is taken as the unit. The set would look as follows:

1:1/9:1/5:2/5:3/5:1/3:1/2:3/4:9/10.

The master of course knew these correlations and for making an amphora of a certain capacity it was sufficient for him to have one measuring tool on which all the necessary dimensions were marked.

The master prepared an amphora of a given capacity by using a template.<f31> In order to make a template it is necessary to calculate the theoretical capacity of the vessel. We have already spoken of the formulas of Heron for calculating the theoretical capacity of pithoi of various forms. Such formulas are not known for calculating the capacity of amphoras. But, as I.B. Brashinskii's research has shown, one can apply the above-mentioned formula of Heron for calculating the capacity of amphoras of pithoid type.<f32> M. Lang and I.B. Brashinskii consider the use of these formulas of Heron for calculating the capacities of all types of pointed amphoras to be impermissible.<f33> However, the introduction of a coefficient into Heron's formula makes it in our opinion acceptable for this purpose. We will explain this proposition.

=== 31. Leipunskaia, "On the Standards ...," p133; Brashinskii, "Methods of Studying ...," p90 f.
=== 32. Brashinskii, "Methods of Studying ...," p92.
=== 33. <i>Ibid<i>. p90--93.

Khersonesan amphoras like the amphoras of many other centres may be divided into geometrical figures: cylinder (neck), truncated cone (the section between the line of the base of the neck and the line of the maximum diameter), and cone (lower part of the amphora from the line of maximum diameter). It is possible to get the capacity of an amphora by adding the sum of the volumes of these figures calculated by the accepted formulas: $V = 1/4\ \pi\ d^{2}_{cyl} \times h_{cyl} + 1/12\ \pi\ h_{trunc\text{-}con} (d^{2}_{cyl} + d_{cyl} \times d_{bod} + d^{2}_{bod}) + 1/12\ \pi\ d^{3}_{bod} \times h_{con}$.

If we use the calculations of the correlation of the height of an amphora to the heights of the individual parts and of the diameter of the body to the diameter of the neck and express in the quoted formula the diameter of the neck and the heights of the individual parts of an amphora by means of these correlations, then the formula would look as follows:
$V = 1/4\ \pi \times 0.24d^{2}_{bod} \times 0.25h_{amph} + 1/12\ \pi \times 0.13h_{amph} \times (0.24d^{2}_{bod} + 0.24d_{bod} \times d_{bod} + d^{2}_{bod}) + 1/12\ \pi\ d^{2}_{bod} \times 0.62h_{amph}$. After calculations we obtain the result: $V = 0.259d^{2}h$.

Now let's take Heron's formula: $V = 11/14 \times ((d_{neck} + d_{bod})^{2})/2 \times h$ and carry out the same steps. The final result is: $V = 0.243d^{2}h$.

Comparing the results of calculations by these formulas we can say that they are approximately equal. This gives a basis for using Heron's formula to calculate the capacity of pointed amphoras.

When transforming the formulas, we substituted the maximum value of the correlations of diameters and heights. If we substitute the minimum value of the correlations in these formulas we'll obtain a new value of the coefficient in the final formula. One can conclude from this that the value of the coefficient depends upon the correlations of the diameters and heights of the different parts of an amphora, in other words upon its proportions. The final formula would look like this: $V = xd^{2}h$.

For the amphoras of Khersonesan production the value of x for each type of amphora is determined by the formula $x = V/D^{2}h$.

Since the value of the coefficient has been obtained on scanty material, it is so far premature to claim its stability both for Khersonesan and for amphoras of other centres. Nevertheless, while attempting to check the results of our calculations on the amphoras of Pontic Heraclea and Sinope, we obtained a satisfactory result, i.e, the capacity of an amphora calculated empirically almost coincides [p148] with the capacity calculated by the quoted formula.<f34> We can practically disregard a difference of 200--300 gm.

=== 34. I am not giving the arithmetical value of the coefficient, since it was obtained on an insufficient quantity of material.

We don't know what the formula for calculating bodies of rotation looked like prior to Heron among the Greeks, but we may suppose that Greeks who knew the mathematics of the Egyptians well,<f35> used their formulas. It is known that the Egyptian formula for the volume of a cylinder was $(8/9)^{2}D^{2}H$ or, in contemporary interpretation $\pi/4\ D^{2}H$. In order to calculate the volume of the body of rotation of a different form, the Egyptians introduced a coefficient into the formula of the volume of a cylinder.<f36> Based on these data, we can say that the formula obtained by us for calculating volumes is applicable to bodies (in this case amphoras) of a form which requires the introduction of a corresponding coefficient to the formula of the volume of a cylinder. In other words our formula may look as follows: $V = x\ \pi/4\ D^{2}H$.

=== 35. M.Ia. Vygodskii, <i>Arithmetic and Algebra in the Ancient World [Arifmetika i algebra v drevnem mire]<i> Moscow 1967 p9.
=== 36. G.P. Boev, "Calculation of Surfaces and Volumes of Bodies of Rotation by the Ancient Egyptians [Vychislenie poverkhnostei i ob"emov tel vrashcheniia u drevnikh egiptian]," <i>VDI<i> 1950 nr 3 p200.

Analysing the material obtained, we can draw the following conclusions:

1) In Khersonesos during the Hellenistic period definite standards of measures of capacity are distinguished. Among them, for end IV--mid-III century BCE the most characteristic was a standard of 19.40--19.60 l., and for mid-III--II BCE a standard of 4.80--5.00 l. The fluctuation of standards is probably explained by a change in the conditions of the economic basis of Khersonesos.<f37>

=== 37. These changes are testified to by political events: loss of possessions in the north-west Crimea by the beginning of the second century BCE, the raids of the barbarians on the urban neighbourhoods of Khersonesos in III--II century BCE, and destruction of the out-lying settlements of potters and vintners.

2) During the same period in Khersonesos one observes a certain unification of the standards of measures. At first glance, the standards seem varied (in construction and architecture the use of the Ionic foot is recorded; in trade, of the Chian-Rhodian and Persian monetary-weight systems; in ceramic production, of Euboian and possibly Pontic standards of capacity). However the weight of a capacity filled with water equal to the weight of the Chian-Rhodian drachma (3.88 gm), comprises 1/10000, of the Persian 1/7000, of a Euboian metretes of 38.88 kg, the metric equivalent of which is 1.5 cubic feet at 296 mm. [sic: sentence ungrammatical---is there an erratum slip?] In other words the standards of capacity and weight may be reduced to a single modulus: the Ionic foot at 296 mm.

3) Comparing the linear parameters of the amphoras from different centres, one may observe a striving towards standardisation of the linear measures of this type of production in certain regions, in particular in Khersonesos and the centres which traded with it: Pontic Heraclea, Sinope, Thasos, and Pantikapaia.

4) The linear dimensions of amphoras exist in a proportional dependency which is determined by the form of the amphora. In the formula this dependancy is expressed by a coefficient.

5) For mathematical calculation of the capacity of pointed amphoras one can apply the formula x(8/9)^{2}D^{2}H, using a certain coefficient for each type of amphora. Such calculations make it possible to determine the capacity of fragmented vessels, which is particularly important for the analysis of ceramic material, the major quantity of which consists of sherds of amphoras.

6) Probably the Greeks strove to make uniform not only the standards of measures but also the methods calculating them, which had a great significance in ancient production and trade.

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